

Math 120 Final, Autumn 2006 — Problem 1

Let p be the price (dollars) and N the tickets sold. N is linear in p with slope $-\frac{45}{1.12} \approx -40.179$ tickets per dollar, and $N(10) = 800$:

$$N(p) = 800 - \frac{45}{1.12}(p - 10) = 1201.786 - 40.179p.$$

Revenue is price times tickets:

$$R(p) = p N(p) = 1201.786p - 40.179p^2,$$

a downward parabola. Its maximum is at the vertex

$$p = \frac{1201.786}{2(40.179)} \approx 14.96.$$

Answer: set the price at about \$14.96 (about 601 tickets, revenue $\approx$ \$8,987).

Math 120 Final, Autumn 2006 — Problem 2

The chain moves at the same linear speed over both sprockets. Front sprocket: $30 \text{ rev/min} = 60\pi \text{ rad/min}$, radius 9 in, so the chain speed is

$$v = 9(60\pi) = 540\pi \text{ in/min.}$$

Rear sprocket (radius 4 in) angular speed: $\omega = \frac{540\pi}{4} = 135\pi \approx 424.1 \text{ rad/min}$. The rear wheel turns with the rear sprocket, so its rim moves at $R\omega = 5500 \text{ in/min}$:

$$R = \frac{5500}{135\pi} \approx \boxed{12.97 \text{ inches}}.$$

Math 120 Final, Autumn 2006 — Problem 3

One lap takes 95 s at 5 m/s, so the circumference is 475 m and the radius is $r = \frac{475}{2\pi} \approx 75.60$ m.

8.5 min = 510 s = 5(95) + 35 s, so after 5 full laps she has run another 35(5) = 175 m along the track from her starting point. That arc subtends

$$\theta = \frac{175}{r} \approx 2.3149 \text{ rad.}$$

The straight-line distance between two points on a circle separated by angle θ is the chord $2r \sin(\theta/2)$:

$$d = 2(75.60) \sin(1.1574) \approx \boxed{138.5 \text{ m}}.$$

(The 91 s fact only places her start just past the northernmost point; it doesn't change this distance.)

Math 120 Final, Autumn 2006 — Problem 4

(a) $g(2x) = -3$ when $2x \leq 10$, i.e. $x \leq 5$, and $g(2x) = \sqrt{2x}$ when $x > 5$. Then $h(u) = u^2 + 4$ gives

$$h(g(2x)) = \begin{cases} (-3)^2 + 4 = 13, & x \leq 5 \\ (\sqrt{2x})^2 + 4 = 2x + 4, & x > 5 \end{cases}$$

(b)

$$\frac{h(3a) - h(2a)}{a} = \frac{(9a^2 + 4) - (4a^2 + 4)}{a} = \frac{5a^2}{a} = \boxed{5a} \quad (a \neq 0).$$

Math 120 Final, Autumn 2006 — Problem 5

Let h be the height and d Fred's first distance from the building. Then

$$h = d \tan 38^\circ \quad \text{and} \quad h = (d + 100) \tan 34^\circ.$$

Setting these equal: $d(\tan 38^\circ - \tan 34^\circ) = 100 \tan 34^\circ$, so

$$d = \frac{100 \tan 34^\circ}{\tan 38^\circ - \tan 34^\circ} \approx 631.70 \text{ ft}, \quad h = d \tan 38^\circ \approx \boxed{493.5 \text{ ft}}.$$

Math 120 Final, Autumn 2006 — Problem 6

Max 52 at $t = 2$, min -108 at $t = 7.6$: half a period is 5.6 h, so the period is 11.2 h. Midline $\frac{52+(-108)}{2} = -28$, amplitude 80. Starting at the max:

$$T(t) = -28 + 80 \cos\left(\frac{2\pi}{11.2}(t - 2)\right).$$

$T < 10 \iff \cos\left(\frac{2\pi}{11.2}(t - 2)\right) < \frac{38}{80} = 0.475$. With $\alpha = \cos^{-1}(0.475) \approx 1.0758$, this happens when $\frac{2\pi}{11.2}(t - 2)$ lies between α and $2\pi - \alpha$ (plus multiples of 2π), i.e. for

$$t \in (3.918, 11.282) \text{ and } (15.118, 22.482) \text{ and (just at the start) } [0, 0.082).$$

Total time below 10°C in the first 24 hours:

$$7.365 + 7.365 + 0.082 \approx \boxed{14.8 \text{ hours}}.$$

Math 120 Final, Autumn 2006 — Problem 7

Newberg: $N(t) = 1325 \cdot 4^{t/15}$ (t in years after Jan 1, 2000). So $N(5) = 1325 \cdot 4^{1/3} \approx 2103.3$ and Kowplunc then had $2N(5) \approx 4206.6$.

Kowplunc grows exponentially: $K(t) = 550 b^t$ with $550 b^5 = 4206.6$, so $b = (7.6484)^{1/5} \approx 1.50215$.

Solve $550 b^t = 20000$:

$$t = \frac{\ln(20000/550)}{\ln b} = \frac{\ln 36.364}{\ln 1.50215} \approx \boxed{8.83 \text{ years after Jan 1, 2000}}.$$

Math 120 Final, Autumn 2006 — Problem 8

(a) Direction from $(10, 15)$ to $(6, -2)$ is $\langle -4, -17 \rangle$, length $\sqrt{305} \approx 17.464$. Moving 2 m/s:

$$x(t) = 10 - \frac{8}{\sqrt{305}}t \approx 10 - 0.458t, \quad y(t) = 15 - \frac{34}{\sqrt{305}}t \approx 15 - 1.947t.$$

(b) Iseult covers $\langle -17, -2 \rangle$ in 8 s:

$$x(t) = 18 - \frac{17}{8}t, \quad y(t) = 3 - \frac{1}{4}t.$$

(c) $x(t) = 0 \Rightarrow t = \frac{8 \cdot 18}{17} = \frac{144}{17} \approx \boxed{8.47 \text{ s}}$ (continuing on the same line past $(1, 1)$, at the point $(0, 0.88)$).