

Math 120 Final, Winter 2009 — Problem 1

Let d be your first distance from the wall and h_1 your friend's first height: $h_1 = d \tan 62^\circ$. At distance $2d$: $h_2 = 2d \tan 46^\circ$ and $h_2 = h_1 + 8$.

$$2d \tan 46^\circ = d \tan 62^\circ + 8 \Rightarrow d = \frac{8}{2 \tan 46^\circ - \tan 62^\circ} \approx 42.03 \text{ ft.}$$

So $h_2 = 2d \tan 46^\circ \approx \boxed{87.0 \text{ ft}}$ (it was ≈ 79.0 ft at the first measurement).

Math 120 Final, Winter 2009 — Problem 2

Center the mud circle at the origin (radius 20). The westernmost point is $(-20, 0)$, so the cat starts at $(-20, -30)$. It heads toward $(20 + 5, 0 + 11) = (25, 11)$, direction $\langle 45, 41 \rangle$: $(x, y) = (-20 + 45s, -30 + 41s)$.

It reaches $y = 0$ (due east of the center) at $s = \frac{30}{41}$, the point $(12.927, 0)$. It entered the mud when $(-20 + 45s)^2 + (-30 + 41s)^2 = 400$:

$$3706s^2 - 4260s + 900 = 0 \Rightarrow s \approx 0.2731, \text{ entry} \approx (-7.446, -18.562).$$

Mud walked on the first leg: distance from $(-7.446, -18.562)$ to $(12.927, 0) \approx 27.56$ m. Then east from $(12.927, 0)$ to the edge $(20, 0)$: 7.07 m.

Time in the mud: $\frac{27.56 + 7.07}{1.3} \approx \boxed{26.6 \text{ s}}$.

Math 120 Final, Winter 2009 — Problem 3

Let t be years after 2000. Bullm: $B(t) = 7000 \cdot 2^{(t-5)/17}$, so $B(11) = 7000 \cdot 2^{6/17} \approx 8940.1$.

Aarb: $A(t) = 5000b^t$ with $A(11) = 2B(11) \approx 17880.3$, so $b = (3.5761)^{1/11} \approx 1.12282$.

Set $A(t) = 40000$: $b^t = 8 \Rightarrow t = \frac{\ln 8}{\ln b} \approx \boxed{17.95 \text{ years after 2000}}$.

Math 120 Final, Winter 2009 — Problem 4

Let t be hours after midnight yesterday. Max 6 cm at $t = 11$, min 5 cm at $t = 20$: half-period 9 h, period 18 h, midline 5.5, amplitude 0.5:

$$L(t) = 5.5 + 0.5 \cos\left(\frac{2\pi}{18}(t - 11)\right).$$

$5.2 < L < 5.6 \iff -0.6 < \cos\left(\frac{\pi}{9}(t - 11)\right) < 0.2$. Solving over $0 \leq t \leq 24$ gives

$$(4.66, 7.08), (14.92, 17.34), (22.66, 24],$$

so the whiskers were in that range for about $2.42 + 2.42 + 1.34 \approx \boxed{6.18 \text{ hours}}$.

Math 120 Final, Winter 2009 — Problem 5

(a) Jerry: $(-5, 5) \rightarrow (10, 0)$ in 5 s: $x_J = -5 + 3t$, $y_J = 5 - t$.

(b) Tom: $(0, 9) \rightarrow (4, -3)$ in 4 s: $x_T = t$, $y_T = 9 - 3t$.

(c) Squared distance:

$$D(t) = (x_J - x_T)^2 + (y_J - y_T)^2 = (2t - 5)^2 + (2t - 4)^2 = 8t^2 - 36t + 41,$$

smallest at $t = \frac{36}{16} = \boxed{2.25 \text{ s}}$ (distance $\frac{1}{\sqrt{2}} \approx 0.71 \text{ m}$).

Math 120 Final, Winter 2009 — Problem 6

At 4 km/h: north 1 km takes $\frac{1}{4}$ h, east 2 km takes $\frac{1}{2}$ h, south 3 km takes $\frac{3}{4}$ h. With the start at the origin:

- $0 \leq t \leq 0.25$: $(0, 4t)$.
- $0.25 \leq t \leq 0.75$: $(4t - 1, 1)$.
- $0.75 \leq t \leq 1.5$: $(2, 1 - 4(t - 0.75)) = (2, 4 - 4t)$.

$$d(t) = \begin{cases} 4t, & 0 \leq t \leq 0.25 \\ \sqrt{(4t - 1)^2 + 1}, & 0.25 < t \leq 0.75 \\ \sqrt{4 + (4 - 4t)^2}, & 0.75 < t \leq 1.5 \end{cases} \quad (\text{km})$$

Math 120 Final, Winter 2009 — Problem 7

(a) $f(g(x)) = (3x - 1) + 2|3x - 4|$.

If $x \geq \frac{4}{3}$: $3x - 1 + 6x - 8 = -4x \Rightarrow x = \frac{9}{13}$, but $\frac{9}{13} < \frac{4}{3}$: rejected.

If $x < \frac{4}{3}$: $3x - 1 - 6x + 8 = -4x \Rightarrow -3x + 7 = -4x \Rightarrow x = -7$ ✓.

Solution: $\boxed{x = -7}$.

(b) $h(x) = g(\sqrt{x}) + x + 1 = 3\sqrt{x} - 1 + x + 1 = x + 3\sqrt{x}$ for $x \geq 0$. Set $y = x + 3\sqrt{x}$ and $u = \sqrt{x} \geq 0$: $u^2 + 3u - y = 0$, so $u = \frac{-3 + \sqrt{9 + 4y}}{2}$. Hence

$$h^{-1}(x) = \left(\frac{-3 + \sqrt{9 + 4x}}{2} \right)^2, \quad x \geq 0.$$