

Math 124 K - Autumn 2009  
Mid-Term Exam Number One  
October 27, 2009  
Answers

1. (a) -4 (b) 1 (c)  $\frac{1}{2\sqrt{3}}$  (d) 1

2. (a)

$$f'(x) = \frac{(12x^11 - 1)(2x^5 + 2 \sin x) - (x^12 - x + 6)(10x^4 + 2 \cos(x))}{(2x^5 + 2 \sin x)^2}$$

(b)

$$g'(x) = (\sec x + x \sec x \tan x)e^x + x \sec x e^x$$

3. Many correct answers are possible. You just need to give two tangent lines which are perpendicular: there are an infinite number of such lines. To every tangent line to this curve, there is a perpendicular one, so you can just pick one, and then find the perpendicular tangent. Answers include

- $y = x - \frac{9}{4}, y = -x - \frac{1}{4}$
- $y = 2x - 4, y = -\frac{1}{2}x - \frac{9}{16}$
- $y = 4x - 9, y = -\frac{1}{4}x - \frac{49}{64}$
- $y = -2x, y = \frac{1}{2}x - \frac{25}{16}$
- $y = x - 2.25, y = -x - 0.25$

4. To show the function is continuous, you need to show that

$$\lim_{x \rightarrow 0} f(x) = f(0).$$

We see that  $f(0) = 0$ .

To evaluate the limit, we note that

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x \sin\left(\frac{1}{x^3}\right).$$

To evaluate this limit, note that

$$1 \leq \sin\left(\frac{1}{x^3}\right) \leq 1$$

and so, for  $x > 0$ ,

$$-x \leq x \sin\left(\frac{1}{x^3}\right) \leq x.$$

Since

$$\lim_{x \rightarrow 0^+} -x = \lim_{x \rightarrow 0^+} x = 0,$$

by the Squeeze Theorem we may conclude that

$$\lim_{x \rightarrow 0^+} x \sin \left( \frac{1}{x^3} \right) = 0.$$

By a similar argument,

$$\lim_{x \rightarrow 0^-} x \sin \left( \frac{1}{x^3} \right) = 0,$$

and so we can conclude that

$$\lim_{x \rightarrow 0} x \sin \left( \frac{1}{x^3} \right) = 0 = f(0)$$

and so  $f$  is continuous at 0.

5. Let  $\epsilon > 0$ . Let  $\delta = \epsilon/3$ . Then, if  $|x - 2| < \delta$ ,

$$|3x + 4 - 10| = |3x - 6| = 3|x - 2| < 3\delta = \epsilon.$$

6. The tangent line to the curve at  $(a, 1/a)$  is

$$y = -\frac{1}{a^2}x + \frac{2}{a}.$$

This line has  $y$ -intercept  $2/a$  and  $x$ -intercept  $2a$ . So the area is

$$\frac{1}{2} \left( \frac{2}{a} \right) 2a = 2.$$