

Math 124I - Winter 2003
Mid-Term Exam Number One
January 30, 2003

Name: _____

Section: _____

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total	70	

- Complete all questions.
- You may use a scientific (non-graphing) calculator during this examination. Other electronic devices are not allowed.
- You may use one hand-written 8.5 by 11 inch page of notes. You can use both sides of the note page.
- Show all work for full credit.
- You have 50 minutes to complete the exam.

1. Decide whether the following statements are true or false and circle the corresponding letter below the statement. You need not show any work (i.e., these are “all or nothing” problems).

If $\cos \theta = \frac{2}{3}$, then $\sin \theta$ must equal $\frac{\sqrt{5}}{3}$.

T F

Polynomials are continuous everywhere that they are defined.

T F

If $\lim_{x \rightarrow 5^+} f(x) = \infty$ and $\lim_{x \rightarrow 5^-} f(x) = \infty$, then $\lim_{x \rightarrow 5} f(x) = \infty$.

T F

$\lim_{x \rightarrow 3} \frac{1}{(x-3)^8} = \infty$.

T F

$\lim_{x \rightarrow 3} \frac{1}{(x-3)^7} = \infty$.

T F

2. Suppose $f(x)$ is defined for all $x \neq 0$, and that $f'(x) = \frac{3}{2}x^2 - \frac{1}{x}$. Find the x-value of each point on the curve $y = f(x)$ at which the tangent line is horizontal.

3. Let

$$f(x) = \begin{cases} x^3 - 5 & \text{if } x < 4 \\ 3x + 47 & \text{if } 4 \leq x \leq 7 \\ x^2 + 29 & \text{if } x > 7 \end{cases}$$

Find all points at which f is discontinuous. Justify your answer.

4. Suppose

$$\frac{f(a+h) - f(a)}{h} = \frac{-3}{(a-3)(a+h-3)}.$$

(a) Find the slope of the secant line through $(2, f(2))$ and $(8, f(8))$.

(b) Find the slope of the tangent line to the curve $y = f(x)$ at the point $(4, f(4))$.

5. Evaluate the limit:

$$\lim_{x \rightarrow \infty} \frac{\sqrt{(1+2x)(2+3x)}}{3+7x}$$

6. The height (in meters) of an arrow shot straight up from the ground after t seconds is given by

$$h(t) = 60t - 4.9t^2.$$

- (a) Find the average velocity of the arrow over the interval $[1, 3]$.
- (b) Find a 1-second interval of time over which the arrow's average velocity is 48.24 meters/second.

7. Let C be the circle with radius 1 centered at the origin. Find all points on C at which the tangent line to C passes through the point $(10,0)$.