

**Math 125 Final, Spring 2026 — Problem 1**

(a) By parts,  $u = \ln x$ ,  $dv = x^3 dx$ :

$$\frac{x^4}{4} \ln x - \int \frac{x^4}{4} \cdot \frac{1}{x} dx = \boxed{\frac{x^4 \ln x}{4} - \frac{x^4}{16} + C}.$$

(b)  $x^2 + 2x + 5 = (x + 1)^2 + 4$ ; write  $x = (x + 1) - 1$ :

$$\int \frac{x + 1}{\sqrt{(x + 1)^2 + 4}} dx - \int \frac{dx}{\sqrt{(x + 1)^2 + 4}} = \boxed{\sqrt{x^2 + 2x + 5} - \ln \left| x + 1 + \sqrt{x^2 + 2x + 5} \right| + C}.$$

**Math 125 Final, Spring 2026 — Problem 2**

(a)  $(1 + x^{1/3})^2 = 1 + 2x^{1/3} + x^{2/3}$ :

$$\int_0^1 = \left[ x + \frac{3}{2}x^{4/3} + \frac{3}{5}x^{5/3} \right]_0^1 = 1 + \frac{3}{2} + \frac{3}{5} = \boxed{\frac{31}{10}}.$$

(b)  $\frac{x^2 - 2x + 3}{x^2(x+1)} = -\frac{5}{x} + \frac{3}{x^2} + \frac{6}{x+1}$  (from  $x = 0$ :  $B = 3$ ;  $x = -1$ :  $C = 6$ ;  $x^2$ -coefficient:  $A + C = 1$ ):

$$\left[ -5 \ln x - \frac{3}{x} + 6 \ln(x+1) \right]_1^2 = -5 \ln 2 + \frac{3}{2} + 6 \ln \frac{3}{2} = \boxed{\frac{3}{2} + 6 \ln 3 - 11 \ln 2} \approx 0.467.$$

**Math 125 Final, Spring 2026 — Problem 3**

Let  $u = \sqrt{x}$ ,  $dx = 2u \, du$ ;  $x = 4 \rightarrow u = 2$ :

$$\int_4^\infty \frac{3 \, dx}{\sqrt{x}(4+x)} = \lim_{t \rightarrow \infty} \int_2^t \frac{6 \, du}{4+u^2} = \lim_{t \rightarrow \infty} \left[ 3 \tan^{-1} \frac{u}{2} \right]_2^t = 3 \left( \frac{\pi}{2} - \frac{\pi}{4} \right) = \boxed{\frac{3\pi}{4}}.$$

**Math 125 Final, Spring 2026 — Problem 4**

(a)  $\int_0^2 f = \int_0^1 f + \int_1^2 f = 2 + 3 = \boxed{5}$ .

(b) With  $u = x^2$ ,  $du = 2x \, dx$ :  $7 = \int_1^2 xf(x^2)dx = \frac{1}{2} \int_1^4 f(u) \, du$ , so  $\int_1^4 f = \boxed{14}$ .

(c) With  $u = x^3$ ,  $du = 3x^2 dx$ :  $10 = \frac{1}{3} \int_1^8 f$ , so  $\int_1^8 f = 30$  and  $\int_4^8 f = 30 - 14 = \boxed{16}$ .

**Math 125 Final, Spring 2026 — Problem 5**

Height above the top of the building:  $h'' = -32$ ,  $h'(t) = v_0 - 32t$ . Passing the window at  $t = 4$  while falling at 96 ft/s:  $v_0 - 128 = -96 \Rightarrow v_0 = 32$ . So  $h(t) = 32t - 16t^2$ .

It hits the ground at  $t = 5$ :  $h(5) = 160 - 400 = -240$ , so the building is 240 ft tall. The window is at  $h(4) = 128 - 256 = -128$ , i.e. 128 ft below the top:

$$\text{window height} = 240 - 128 = \boxed{112 \text{ ft}}.$$

Math 125 Final, Spring 2026 — Problem 6

Let  $z$  be height measured from the tank’s center ( $-2 \leq z \leq 2$ ). The center is  $1 + 2 = 3$  m below ground, so a slice at  $z$  is lifted  $3 - z$  m. Its area is  $\pi(4 - z^2)$ :

$$W = \int_{-2}^2 9800\pi(4 - z^2)(3 - z) \, dz = 9800\pi \left[ 3 \int_{-2}^2 (4 - z^2) \, dz - \underbrace{\int_{-2}^2 z(4 - z^2) \, dz}_{0 \text{ (odd)}} \right] = 9800\pi \cdot 3 \cdot \frac{32}{3}.$$

$$W = 313600\pi \approx \boxed{9.85 \times 10^5 \text{ J}}.$$

**Math 125 Final, Spring 2026 — Problem 7**

On  $[0, \frac{2\pi}{3}]$ :  $\cos x - \cos 2x = -(2 \cos x + 1)(\cos x - 1) \geq 0$ , so  $\cos x$  is on top.

(a)  $A = \left[ \sin x - \frac{\sin 2x}{2} \right]_0^{2\pi/3} = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} = \boxed{\frac{3\sqrt{3}}{4}}.$

(b)  $M_y = \int_0^{2\pi/3} x(\cos x - \cos 2x)dx = \left[ x \sin x + \cos x - \frac{x \sin 2x}{2} - \frac{\cos 2x}{4} \right]_0^{2\pi/3} = \frac{\pi\sqrt{3}}{2} - \frac{9}{8}.$

$$M_x = \int_0^{2\pi/3} \frac{1}{2}(\cos^2 x - \cos^2 2x)dx = \frac{1}{4} \int_0^{2\pi/3} (\cos 2x - \cos 4x)dx = \frac{1}{4} \left[ \frac{\sin 2x}{2} - \frac{\sin 4x}{4} \right]_0^{2\pi/3} = -\frac{3\sqrt{3}}{32}.$$

$$(\bar{x}, \bar{y}) = \left( \frac{M_y}{A}, \frac{M_x}{A} \right) = \left( \frac{2\pi}{3} - \frac{\sqrt{3}}{2}, -\frac{1}{8} \right) \approx (1.228, -0.125).$$

Math 125 Final, Spring 2026 — Problem 8

Separate:  $e^{-7y}dy = 2e^{-t}dt \Rightarrow -\frac{1}{7}e^{-7y} = -2e^{-t}+C$ .  $y(0) = 0$ :  $-\frac{1}{7} = -2+C$ , so  $C = \frac{13}{7}$ . Then  $e^{-7y} = 14e^{-t}-13$ :

$$y(t) = -\frac{1}{7}\ln(14e^{-t} - 13).$$

**Math 125 Final, Spring 2026 — Problem 9**

**(a)**  $\frac{dA}{dt} = k\sqrt{A}(100 - A).$

**(b)** Let  $u = \sqrt{A}$  ( $A = u^2$ ,  $A' = 2uu'$ ):  $2uu' = ku(100 - u^2)$ , so  $u' = \frac{k}{2}(100 - u^2)$ . Separate with  $\frac{1}{100-u^2} = \frac{1}{20}\left(\frac{1}{10-u} + \frac{1}{10+u}\right)$ :

$$\frac{1}{20} \ln \frac{10+u}{10-u} = \frac{k}{2}t + C \Rightarrow \frac{10+u}{10-u} = Be^{10kt}.$$

$A(0) = 4$  ( $u = 2$ ):  $B = \frac{12}{8} = \frac{3}{2}$ .  $A(9) = 25$  ( $u = 5$ ):  $3 = \frac{3}{2}e^{90k} \Rightarrow e^{10k} = 2^{1/9}$ . So with  $R(t) = \frac{3}{2} \cdot 2^{t/9}$ ,  $u = 10^{\frac{R-1}{R+1}}$ :

$$A(t) = 100\left(\frac{\frac{3}{2}2^{t/9} - 1}{\frac{3}{2}2^{t/9} + 1}\right)^2 = 100\left(\frac{3 \cdot 2^{t/9} - 2}{3 \cdot 2^{t/9} + 2}\right)^2$$
(check:  $A(0) = 4$ ,  $A(9) = 25$ ).