

Math 125 A and B Midterm 1 Spring 2024 Solutions

Prof. Charles Camacho

Math 125 A & B

Midterm 2 Exam, Spring 2024

Print Your Full Name

Solutions

Signature

Student ID Number

Quiz Section

Instructor's Name

TA's Name

Please read these instructions!

1. Your exam contains 8 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The exam is worth 50 points. Point values for problems vary and these are clearly indicated. You have 50 minutes for this exam.
4. Unless otherwise stated, make sure to SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. Write your final answer in the provided location for each problem. Unless otherwise indicated, final answers do not need to be simplified.
6. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π , $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form. Note that trigonometric functions on the unit circle, such as $\sin(\pi/6)$ and $\tan(\pi/3)$, must be written in exact form (e.g., $\sin(\pi/6) = 1/2$).
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit may be awarded for use of methods that are learned later in the course or from other (non-prerequisite) courses.

Problem	Total Points
1	10
2	10
3	10
4	10
5	10
Total	50

1. Evaluate the following integrals. If you do a substitution, you must show that you know how to appropriately change bounds. Do not simplify your final answer.

(a) (5 points) $\int \frac{3x^2 - 2x + 1}{\sqrt{x}} dx$

$$= \int \left(\frac{3x^2}{\sqrt{x}} - \frac{2x}{\sqrt{x}} + \frac{1}{\sqrt{x}} \right) dx$$

$$= \int \left(3x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx$$

$$= \boxed{3 \cdot \frac{2x^{\frac{5}{2}}}{5} - 2 \cdot \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C}$$

(b) (5 points) $\int_0^2 (4x+1)^{2/3} dx$

$$u = 4x+1$$

$$du = 4 dx$$

$$\frac{1}{4} du = dx$$

$$x=0 \rightarrow u=1$$

$$x=2 \rightarrow u=9$$

$$= \int_1^9 u^{\frac{2}{3}} \cdot \frac{1}{4} du$$

$$= \frac{1}{4} \left[\frac{3}{5} u^{\frac{5}{3}} \right]_1^9$$

$$= \boxed{\frac{3}{20} \left(9^{\frac{5}{3}} - 1 \right)}$$

2. A motorist applies the brakes on a car, causing an acceleration of $a(t) = -16t$ ft/sec² at time t in seconds since applying the brakes. Assume the car has a velocity of 32 ft/sec when brakes are first applied.

(a) (3 points) Find the velocity $v(t)$ of the car at time t in seconds since brakes were applied.

$$\begin{aligned} v(t) &= \int a(t) dt & 32 &= v(0) = 0 + C \rightarrow C = 32 \\ &= \int -16t dt & \Rightarrow & \boxed{v(t) = -8t^2 + 32} \\ &= -8t^2 + C \end{aligned}$$

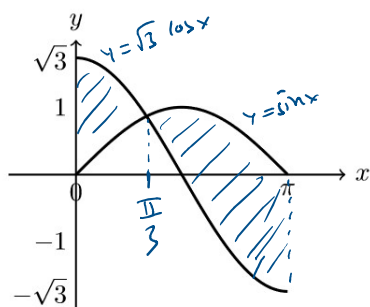
(b) (3 points) How many seconds does it take for the car to stop after brakes were applied?

$$\begin{aligned} -8t^2 + 32 &= 0 & \boxed{t = 2 \text{ seconds}} \\ t^2 &= 4 \\ t &= \pm 2 \end{aligned}$$

(c) (4 points) How far does the car travel after brakes were applied?

$$\begin{aligned} \text{distance} &= \int_0^2 |v(t)| dt = \int_0^2 (-8t^2 + 32) dt \\ &= \left[-\frac{8t^3}{3} + 32t \right]_0^2 \\ &= -\frac{64}{3} + 64 = \frac{2}{3} \cdot 64 = \boxed{\frac{128}{3} \text{ ft}} \end{aligned}$$

3. (10 points) Find the area in the xy -plane bounded by the graphs of $y = \sin x$ and $y = \sqrt{3} \cos x$ between $x = 0$ and $x = \pi$ (see the figure below).



$$\begin{aligned} \text{Area} &= \int_0^{\frac{\pi}{3}} (\sqrt{3} \cos x - \sin x) dx + \int_{\frac{\pi}{3}}^{\pi} (\sin x - \sqrt{3} \cos x) dx \\ &= \left[\sqrt{3} \sin x + \cos x \right]_0^{\frac{\pi}{3}} + \left[-\cos x - \sqrt{3} \sin x \right]_{\frac{\pi}{3}}^{\pi} \end{aligned}$$

Intersection:

$$\sin x = \sqrt{3} \cos x$$

$$\tan x = \sqrt{3}$$

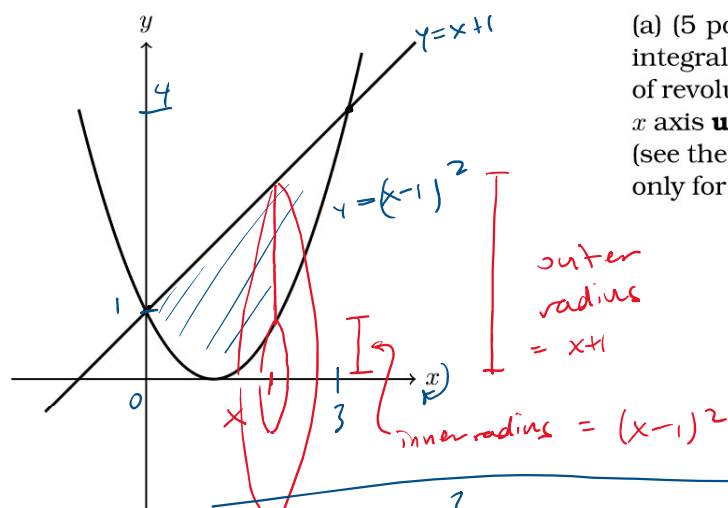
$$x = \frac{\pi}{3}$$

$$= \sqrt{3} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} - (0 + 1) + \left[1 - 0 - \left(-\frac{1}{2} - \sqrt{3} \cdot \frac{\sqrt{3}}{2} \right) \right]$$

$$= \frac{3}{2} + \frac{1}{2} - 1 + 1 + \frac{1}{2} + \frac{3}{2}$$

$$= \boxed{4}$$

4. Let R be the region in the xy -plane bounded above by the graph of $y = x + 1$ and below by $y = (x - 1)^2$.



- (a) (5 points) **Set up, but do not evaluate**, an integral that computes the volume of the solid of revolution generated by revolving R about the x axis **using the method of disks and washers** (see the figure to the left). Full credit is awarded only for using the method of disks and washers.

Intersection:

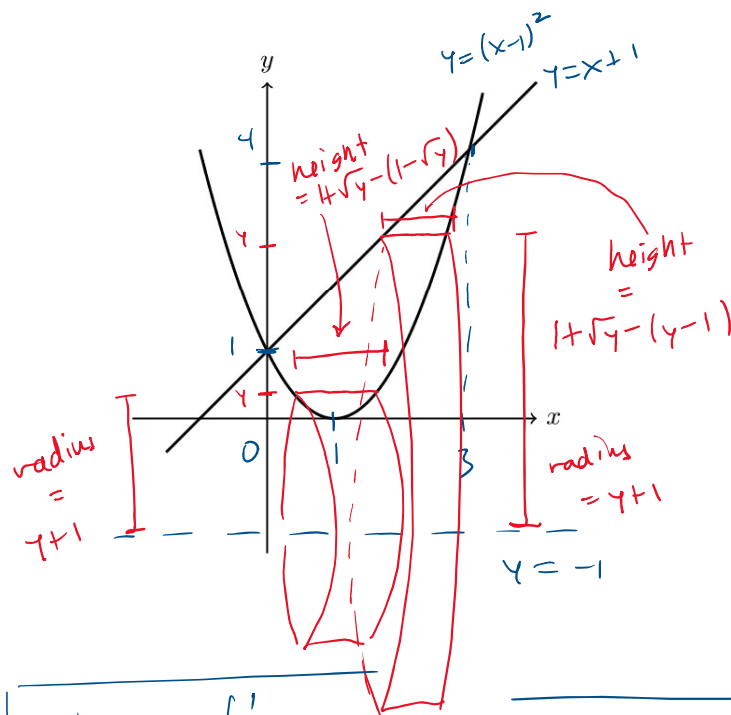
$$x + 1 = (x - 1)^2$$

$$x + 1 = x^2 - 2x + 1$$

$$0 = x^2 - 3x$$

$$0 = x(x - 3) \Rightarrow x = 0, 3$$

$$\text{Volume} = \int_0^3 \left(\pi (x + 1)^2 - \pi ((x - 1)^2)^2 \right) dx$$



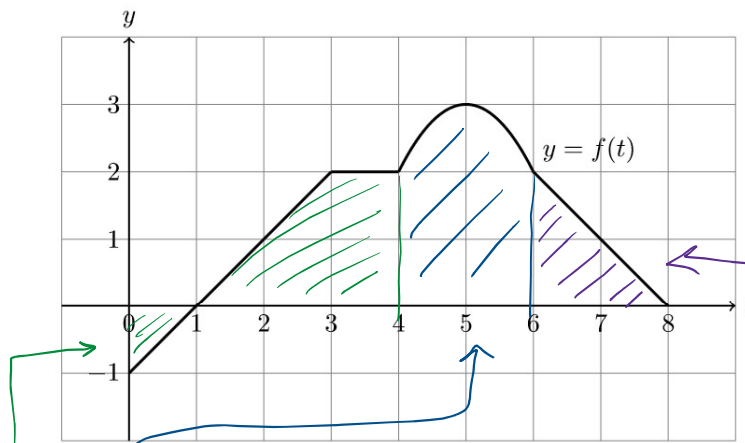
- (b) (5 points) **Set up, but do not evaluate**, an integral that computes the volume of the solid of revolution generated by revolving R about the horizontal line $y = -1$ **using the method of cylindrical shells** (see the figure to the left). Full credit is awarded only for using the method of cylindrical shells.

$$\begin{aligned} y &= x + 1 & y &= (x - 1)^2 \\ y - 1 &= x & \pm \sqrt{y} &= x - 1 \\ 1 \pm \sqrt{y} &= x \end{aligned}$$

$$\text{Volume} = \int_0^1 2\pi (y + 1) (1 + \sqrt{y} - (1 - \sqrt{y})) dy + \int_1^4 2\pi (y + 1) (1 + \sqrt{y} - (y - 1)) dy$$

5. **For this problem, you do not need to show your work.** Answer the following questions based on the graph of $y = f(t)$ below, which has the domain $0 \leq t \leq 8$.

The function $y = f(t)$ consists of a line for $0 \leq t \leq 3$; a horizontal line between $3 < t \leq 4$; an upper semicircle of radius 1 between $4 < t < 6$; and a line between $6 \leq t \leq 8$.



(a) (2 points) $\int_0^4 f(t) dt = -\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) + 1 \cdot 2 = -\frac{1}{2} + 2 + 2 = \boxed{\frac{7}{2}}$

(b) (2 points) $\int_2^3 f(2t) dt = \int_4^6 f(u) \cdot \frac{1}{2} du = \frac{1}{2} \left(\frac{1}{2} \pi (1)^2 + 2 \cdot 2 \right) = \boxed{\frac{\pi}{4} + 2}$
 $t=2 \rightarrow u=4$ $u=2t$ $\frac{1}{2} du = dt$
 $t=3 \rightarrow u=6$ $du = 2 dt$

(c) (2 points) $\int_1^6 f'(t) dt = f(6) - f(1) = 2 - 0 = \boxed{2}$

(d) (2 points) Let $g(x) = \int_2^{x^2} (t + f(t)) dt$. Then $g'(2) = (4 + f(4)) \cdot 4 = (4 + 2) \cdot 4 = \boxed{24}$

$$g'(x) = (x^2 + f(x^2)) \cdot 2x$$

(e) (2 points) $\lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(6 + \frac{2i}{n}\right) \cdot \frac{2}{n} = \int_6^8 f(x) dx = 2 \cdot 2 \cdot \frac{1}{2} = \boxed{2}$

$$\Delta x = \frac{b-a}{n} = \frac{2}{n} \Rightarrow b-a=2$$

$$x_i = a + i\Delta x = 6 + \frac{2i}{n}$$

$$\Rightarrow a = 6$$

$$\Rightarrow b = 8$$

Extra scratch paper.

Extra scratch paper.