

Math 125 A Winter 2025 Midterm 1 Solutions

Prof. Charles Camacho

Math 125 A

Midterm 1 Exam, Winter 2025

Print Your Full Name

Solutions

Signature

Student ID Number

Quiz Section

Instructor's Name

TA's Name

Please read these instructions!

1. Your exam contains 8 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam.
3. The exam is worth 50 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.
4. Unless otherwise stated, make sure to SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. For problems that aren't sketches, place a box around your final answer to each question.
6. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
7. Unless otherwise instructed, always give your answers in exact form. For example, 3π , $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form.
8. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit is awarded for use of methods that are learned later in the course or from other courses.

Problem	Total Points
1	10
2	10
3	10
4	10
5	10
Total	50

1. Evaluate the following integrals. If you do a substitution, you must show that you know how to appropriately change bounds. Do not simplify your final answer.

(a) (5 points) $\int_{-1}^0 35x(x+1)^{3/2} dx$

$u = x+1 \rightarrow x = u-1$

$du = dx$

$= 35 \int_{\boxed{0}}^{\boxed{1}} (u-1) u^{3/2} du$

$= 35 \int_0^1 (u^{5/2} - u^{3/2}) du$

$= 35 \left[\frac{2u^{7/2}}{7} - \frac{2u^{5/2}}{5} \right]_0^1 = \boxed{35 \left[\frac{2}{7} - \frac{2}{5} \right]}$

(b) (5 points) $\int \frac{4ax + 3x^2}{\sqrt{6ax^2 + 3x^3}} dx$, where a is a constant

$u = 6ax^2 + 3x^3$

$du = (12ax + 9x^2) dx$

$\frac{1}{3} du = (4ax + 3x^2) dx$

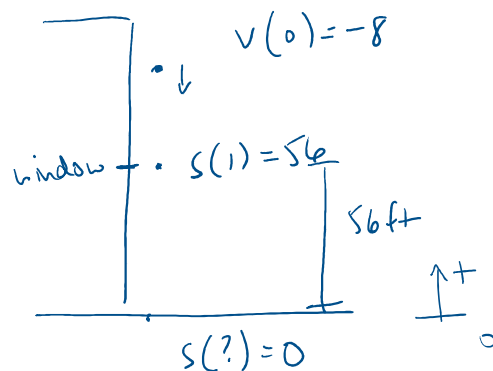
$= \int \frac{\frac{1}{3}}{\sqrt{u}} du$

$= \frac{1}{3} \int u^{-1/2} du$

$= \frac{1}{3} \left[2u^{1/2} \right] + C$

$= \boxed{\frac{2}{3} (6ax^2 + 3x^3)^{1/2} + C}$

2. (10 points) While studying at a window for a calculus exam in a building on the UW Seattle campus, you look up to see that someone on a floor above you threw a tomato downward outside the window with an initial velocity of -8 ft/sec (assume that the upward direction is positive and ignore air friction). Suppose your window is located 56 feet above the ground and the tomato passes your window 1 second after being thrown. Using the acceleration of -32 ft/sec² from gravity, find the time that the tomato hits the ground after being thrown.



$$a(t) = -32$$

$$v(t) = -32t + C$$

$$-8 = v(0) = -32(0) + C \rightarrow C = -8$$

$$v(t) = -32t - 8 \quad (+3)$$

$$s(t) = -16t^2 - 8t + D$$

position

$$56 = s(1) = -16(1)^2 - 8(1) + D$$

$$56 = -24 + D$$

$$80 = D$$

$$\Rightarrow s(t) = -16t^2 - 8t + 80 \quad (+3)$$

$$-16t^2 - 8t + 80 = 0 \quad (+2)$$

$$2t^2 + t - 10 = 0$$

$$(2t + 5)(t - 2) = 0$$

$$\downarrow$$

$$t = -\frac{5}{2}$$

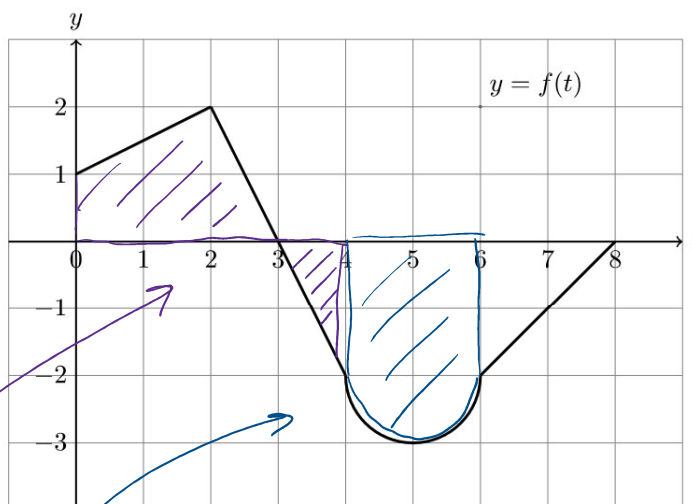
$$\downarrow$$

$$t = 2 \quad (+2)$$

$\therefore$ The tomato hits the ground after 2 seconds.

3. **For this problem, you do not need to show your work.** Answer the following questions based on the graph of $y = f(t)$ below, which has the domain $0 \leq t \leq 8$.

The function $y = f(t)$ consists of a line for $0 < t < 2$; a line between $2 < x < 4$; a lower semicircle of radius 1 between $4 < t < 6$; and a line between $6 < t < 8$.



(a) (2 points) $\int_4^6 f(t) dt = \boxed{-4 + \frac{1}{2}\pi(1)^2}$ (+2)

(b) (2 points) $\int_0^3 f'(t) dt = f(3) - f(0) = 0 - (1) = \boxed{-1}$ (+2)

(c) (2 points) $\int_0^2 2t f(t^2) dt = \int_0^4 f(u) du = 2 \cdot 1 + \frac{1}{2} \cdot 2 \cdot 1 = 2 + 1 = \boxed{3}$ (+2)

$u = t^2$
 $du = 2t dt$

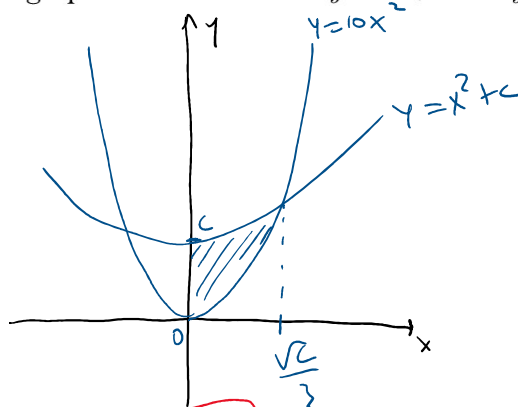
(d) (2 points) Let $g(x) = \int_{x^2}^3 f(t) dt$. Then $g'(2) = -f(4) \cdot 4 = -(-2) \cdot 4 = \boxed{8}$ (+2)

$= -\int_{x^2}^{x^2} f(t) dt$ $g'(x) = -f(x^2) \cdot 2x$

(e) (2 points) $\lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(2 + \frac{4i}{n}\right) \cdot \frac{4}{n} = \int_2^6 f(x) dx = \boxed{-4 + \frac{1}{2}\pi(1)^2}$ (+2)

$x_i = 2 + \frac{4i}{n}$
 $\Delta x = \frac{4}{n}$
 $4 = b - a \rightarrow b = 6$

4. (10 points) Let c be a positive constant. Find the value of c such that the area bounded by the graphs of the functions $y = x^2 + c$ and $y = 10x^2$ in the first quadrant equals 6.



$$10x^2 = x^2 + c$$

$$9x^2 = c$$

$$x^2 = \frac{c}{9}$$

$$x = \pm \frac{\sqrt{c}}{3}$$

first quadrant

$$\rightarrow x = \frac{\sqrt{c}}{3} \quad (+2)$$

$$b = \int_0^{\frac{\sqrt{c}}{3}} (x^2 + c - 10x^2) dx \quad (+2)$$

$$= \int_0^{\frac{\sqrt{c}}{3}} (-9x^2 + c) dx$$

$$= \left[-3x^3 + cx \right]_0^{\frac{\sqrt{c}}{3}} \quad (+2)$$

$$b = -3 \left(\frac{\sqrt{c}}{3} \right)^3 + c \cdot \frac{\sqrt{c}}{3}$$

$$b = -\frac{c^{3/2}}{9} + \frac{c^{3/2}}{3}$$

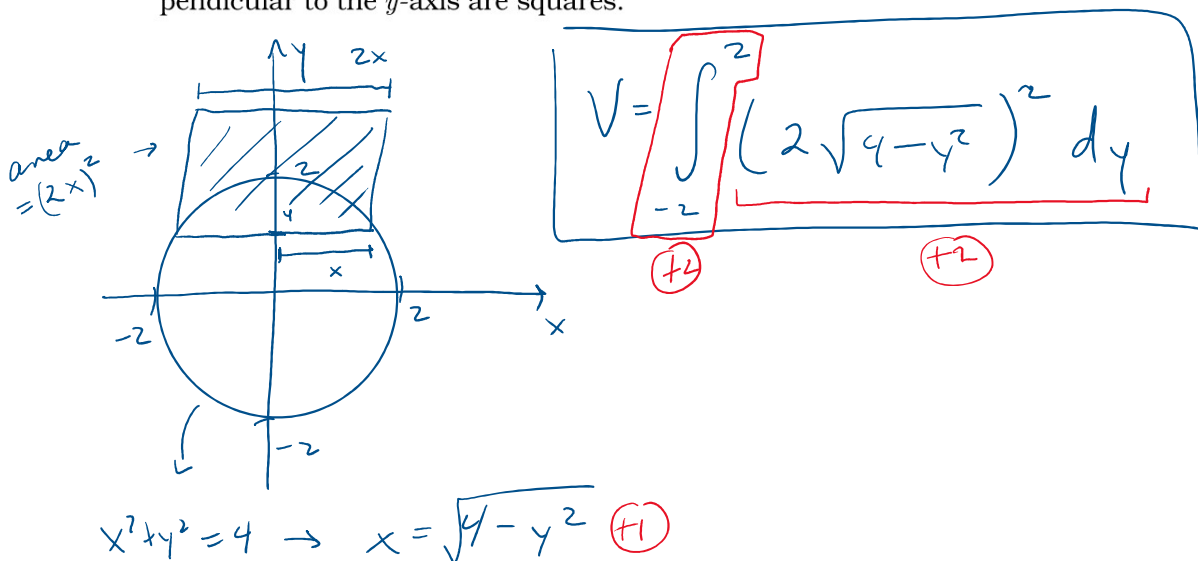
$$b = \frac{2c^{3/2}}{9}$$

$$27 = c^{3/2}$$

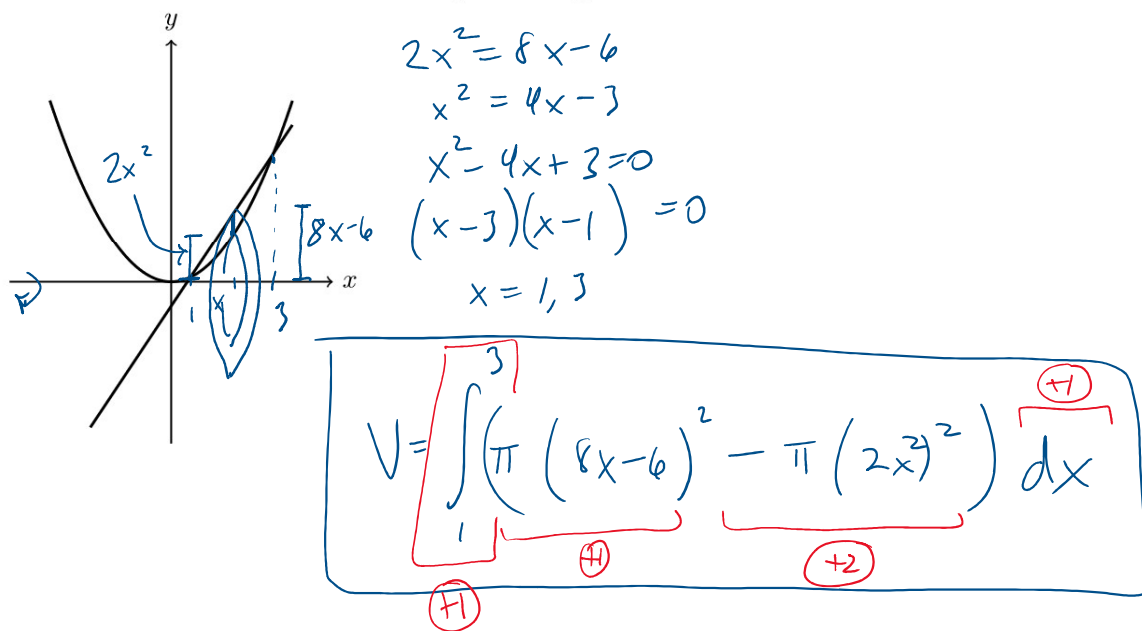
$$(27)^{2/3} = c \rightarrow \boxed{c = 9} \quad (+2)$$

5. (Parts (a) and (b) are unrelated.)

- (a) (5 points) **Set up, but do not evaluate**, an integral equal to the volume of the solid whose base consists of the circular region bounded by $x^2 + y^2 = 4$ and whose cross-sections perpendicular to the y -axis are squares.



- (b) (5 points) **Set up, but do not evaluate**, an integral equal to the volume of the solid of revolution generated by revolving the region bounded by the curves $y = 2x^2$ and $y = 8x - 6$ about the x axis **using the method of disks and washers** (see the figure below, not drawn to scale). Full credit is awarded only for using the method of disks and washers.



Extra scratch paper.

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