

Math 125 Midterm 2, Winter 2016 (Duchamp) — Problem 1

(a) Disks about the x -axis:

$$V = \int_1^5 \pi (e^{-x^2})^2 dx = \int_1^5 \pi e^{-2x^2} dx.$$

(b) $\Delta x = 1$, nodes $1, \dots, 5$, $g(x) = \pi e^{-2x^2}$:

$$S_4 = \frac{1}{3} [g(1) + 4g(2) + 2g(3) + 4g(4) + g(5)] = \frac{\pi}{3} [e^{-2} + 4e^{-8} + 2e^{-18} + 4e^{-32} + e^{-50}] \approx \boxed{0.14}.$$

(The true value is about 0.09: Simpson's rule with $n = 4$ is coarse here because the integrand drops so fast.)

Math 125 Midterm 2, Winter 2016 (Duchamp) — Problem 2

(a) Let $w = \tan^{-1} x$, $dw = \frac{dx}{1+x^2}$:

$$\int \ln w \, dw = w \ln w - w + C = \boxed{\tan^{-1} x \ln(\tan^{-1} x) - \tan^{-1} x + C}.$$

(b) Let $u = \sin 2x$, $du = 2 \cos 2x \, dx$, u from -1 to 1 :

$$\frac{1}{2} \int_{-1}^1 \sqrt{4-u^2} \, du = \frac{1}{2} \left[\frac{u}{2} \sqrt{4-u^2} + 2 \sin^{-1} \frac{u}{2} \right]_{-1}^1 = \frac{1}{2} \left[\sqrt{3} + \frac{2\pi}{3} \right] = \boxed{\frac{\sqrt{3}}{2} + \frac{\pi}{3}}.$$

Math 125 Midterm 2, Winter 2016 (Duchamp) — Problem 3

$\frac{2x+6}{(x-1)(x^2+2x+5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+5}$. At $x=1$: $A = \frac{8}{8} = 1$. Then $2x+6 = (x^2+2x+5) + (Bx+C)(x-1)$ gives $B=-1, C=-1$:

$$\int \left(\frac{1}{x-1} - \frac{x+1}{(x+1)^2+4} \right) dx = \boxed{\ln|x-1| - \frac{1}{2} \ln(x^2+2x+5) + C}.$$

Math 125 Midterm 2, Winter 2016 (Duchamp) — Problem 4

(a) $x' = -e^{-t} \cos 4\pi t - 4\pi e^{-t} \sin 4\pi t$ and $y' = -e^{-t} \sin 4\pi t + 4\pi e^{-t} \cos 4\pi t$. The cross terms cancel:

$$(x')^2 + (y')^2 = e^{-2t}(1 + 16\pi^2), \quad L = \int_0^\infty \sqrt{1 + 16\pi^2} e^{-t} dt.$$

(b) It converges: $\sqrt{1 + 16\pi^2} \lim_{T \rightarrow \infty} [-e^{-t}]_0^T = \boxed{\sqrt{1 + 16\pi^2}} \approx 12.61$.

Math 125 Midterm 2, Winter 2016 (Duchamp) — Problem 5

Measure y up from the tip. By similar triangles the radius at height y is $\frac{y}{2}$. A slice has volume $\pi \frac{y^2}{4} dy$ and is lifted $2 - y$ m:

$$W = \int_0^2 9800\pi \frac{y^2}{4} (2 - y) dy = 2450\pi \left[\frac{2y^3}{3} - \frac{y^4}{4} \right]_0^2 = 2450\pi \cdot \frac{4}{3} = \frac{9800\pi}{3} \approx \boxed{10262.5 \text{ J}}.$$