

Print Your Full Name

Signature

Student ID Number

Quiz Section

Instructor's Name

TA's Name

Please read these instructions!

1. Your exam contains 7 pages; PLEASE MAKE SURE YOU HAVE A COMPLETE EXAM.
2. You are allowed a single, double-sided 8.5"x11" handwritten notesheet; the TI-30XIIS calculator; a writing utensil; and an eraser to use on the exam. No electronic devices are permitted during the exam.
3. The exam is worth 50 points. Point values for problems vary and these are clearly indicated. You have 80 minutes for this exam.
4. Unless otherwise stated, make sure to SHOW YOUR WORK CLEARLY. Credit is awarded to work which is clearly shown and legible. Full credit may not be awarded if work is unclear or illegible.
5. Unless otherwise indicated, please write your final answer in the provided location for each problem. Final answers do not need to be simplified, unless otherwise stated. If you need extra space, use the last two pages of the exam. Clearly indicate on that there is more work located on the last pages, and indicate on those pages the related problem number.
6. Unless otherwise instructed, always give your answers in exact form. For example, 3π , $\sqrt{2}$, and $\ln(2)$ are in exact form; the corresponding approximations 9.424778, 1.4142, and 0.693147 are NOT in exact form. Note that trigonometric functions on the unit circle, such as $\sin(\pi/6)$ and $\tan(\pi/3)$, must be written in fully simplified form (e.g., $\sin(\pi/6)$ is not simplified).
7. Credit is awarded for correct use of techniques or methods discussed in class thus far. Partial credit may be awarded as earned. No credit may be awarded for use of methods that are learned later in the course or from other (non-prerequisite) courses.

Table of Integration Formulas Constants of integration have been omitted.

$$1. \int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$$

$$2. \int \frac{1}{x} dx = \ln|x|$$

$$3. \int e^x dx = e^x$$

$$4. \int b^x dx = \frac{b^x}{\ln b}$$

$$5. \int \sin x dx = -\cos x$$

$$6. \int \cos x dx = \sin x$$

$$7. \int \sec^2 x dx = \tan x$$

$$8. \int \csc^2 x dx = -\cot x$$

$$9. \int \sec x \tan x dx = \sec x$$

$$10. \int \csc x \cot x dx = -\csc x$$

$$11. \int \sec x dx = \ln|\sec x + \tan x|$$

$$12. \int \csc x dx = \ln|\csc x - \cot x|$$

$$13. \int \tan x dx = \ln|\sec x|$$

$$14. \int \cot x dx = \ln|\sin x|$$

$$17. \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

$$18. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$$

Problem	Total Points
1	12
2	8
3	8
4	10
5	12
Total	50

1. Evaluate the following integrals. Fully simplify any trigonometric and logarithmic expressions in your final answer. You do not need to simplify any other algebraic expressions in your final answer.

(a) (6 points) $\int_1^4 3x^2 \ln(x) dx$

$$\begin{aligned}
 &= \left[\ln(x) \cdot x^3 \right]_1^4 - \int_1^4 x^3 \cdot \frac{1}{x} dx \\
 &= \ln(4) \cdot 4^3 - 0 - \int_1^4 x^2 dx \\
 &= \ln(4) \cdot 4^3 - \left[\frac{x^3}{3} \right]_1^4 \\
 &= \boxed{\ln(4) \cdot 4^3 - \frac{4^3}{3} + \frac{1}{3}}
 \end{aligned}$$

IBP:
 $u = \ln(x)$
 $du = \frac{1}{x} dx$
 $v = x^3$
 $dv = 3x^2 dx$

(b) (6 points) $\int \frac{\sqrt{x^2 - 1}}{x} dx$

Trig sub: $x = \sec \theta$
 $dx = \sec \theta \cdot \tan \theta d\theta$

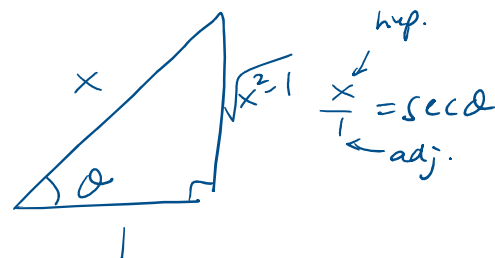
$$\begin{aligned}
 &= \int \frac{\sqrt{\sec^2 \theta - 1}}{\sec \theta} \cdot \sec \theta \cdot \tan \theta d\theta \\
 &= \int \tan^2 \theta d\theta
 \end{aligned}$$

$$= \int \tan^2 \theta d\theta$$

$$= \int (\sec^2 \theta - 1) d\theta$$

$$= \tan \theta - \theta + C$$

$$= \boxed{\sqrt{x^2 - 1} - \operatorname{arccsc}(x) + C}$$



2. (8 points) Evaluate the following integral. Fully simplify any trigonometric and logarithmic expressions in your final answer. You do not need to simplify any other algebraic expressions in your final answer.

$$\int \frac{4}{x^3 + x^2} dx$$

Partial Fractions:

$$\frac{4}{x^3 + x^2} = \frac{4}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$4 = Ax(x+1) + B(x+1) + Cx^2$$

$$4 = x^2(A+C) + x(A+B) + B$$

$$\Rightarrow \begin{aligned} A+C &= 0 \rightarrow C = -A \\ A+B &= 0 \rightarrow A = -B \\ B &= 4 \end{aligned}$$

$$= \int \left(-\frac{4}{x} + \frac{4}{x^2} + \frac{4}{x+1} \right) dx$$

$$= \boxed{-4 \ln|x| - \frac{4}{x} + 4 \ln|x+1| + C}$$

3. (8 points) Evaluate the following improper integral. You must use limits and show your work to receive full credit. If it is convergent, evaluate it. If it diverges, write DIVERGES.

$$\int_1^{\infty} e^{-4x} dx$$

$$= \lim_{t \rightarrow \infty} \int_1^t e^{-4x} dx$$

$$= \lim_{t \rightarrow \infty} \left[-\frac{1}{4} e^{-4x} \right]_1^t$$

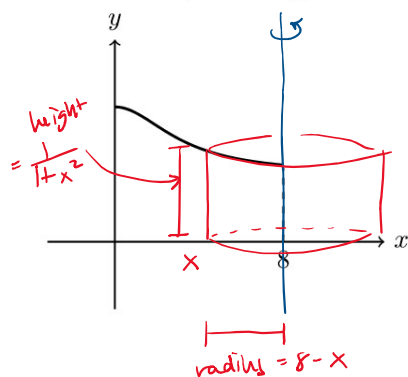
$$= \lim_{t \rightarrow \infty} \left[-\frac{1}{4} e^{-4t} + \frac{1}{4} e^{-4} \right]$$

$$= \lim_{t \rightarrow \infty} \left[\boxed{-\frac{1}{4e^{4t}}} + \frac{1}{4e^4} \right] = \boxed{\frac{1}{4e^4}}$$

↘ 0

4. Consider the region R bounded by the curves $y = \frac{1}{1+x^2}$, $y = 0$, $x = 0$ and $x = 8$. (See figure below.)

- (a) (5 points) Set up, but do not evaluate, an integral equal to the volume of the solid of revolution generated by revolving the region R about the line $x = 8$ using the method of cylindrical shells. Full credit is awarded only for using the method of cylindrical shells.



$$V = \int_0^8 2\pi (8-x) \left(\frac{1}{1+x^2} \right) dx$$

- (b) (5 points) Approximate your integral from part (a) using Simpson's rule with $n = 4$. Round your answer to two decimal places.

$$V = \int_0^8 2\pi (8-x) \cdot \underbrace{\left(\frac{1}{1+x^2} \right)}_{=f(x)} dx$$

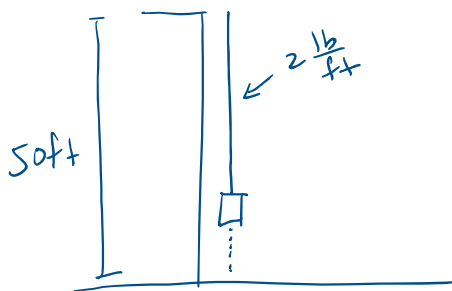
$$\Delta x = \frac{8-0}{4} = 2 \quad \begin{array}{c} | \quad | \quad | \quad | \\ 0 \quad 2 \quad 4 \quad 6 \quad 8 \end{array}$$

$$\approx S_4 = \frac{2}{3} \left(f(0) + 4f(2) + 2f(4) + 4f(6) + f(8) \right)$$

$$\approx \boxed{56.49}$$

5. (12 points) A 50 ft rope weighing 2 lb/ft is lifting a 5 lb bucket initially carrying 8 lb of water. It is carrying the water up a well that is also 50 ft. The bucket of water is being lifted up at a rate of 3 ft/sec. The bucket has a hole on the bottom, causing water to leak out at a rate of 1/4 lb/sec. Find the work required to lift the rope and leaky bucket of water up to the top of the well.

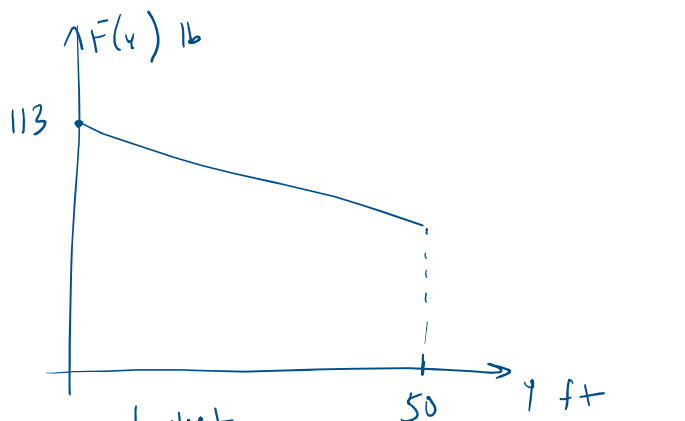
[Hint: find the total weight $F(y)$ in pounds of the rope and leaky bucket as a function of distance y from the bottom of the well.]



$$\frac{\frac{1}{4} \text{ lb}}{1 \text{ sec}} \cdot \frac{1 \text{ sec}}{3 \text{ ft}} = \frac{\frac{1}{4} \text{ lb}}{3 \text{ ft}} = \frac{1}{12} \frac{\text{lb}}{\text{ft}}$$

Initial total weight:

$$F(0) = 50 \text{ ft} \cdot \frac{2 \text{ lb}}{\text{ft}} + 5 \text{ lb} + 8 \text{ lb} = 113 \text{ lb}$$



$$F(y) = \overbrace{50 \text{ lb} \cdot \frac{2 \text{ lb}}{\text{ft}} - \frac{2 \text{ lb}}{\text{ft}} \cdot y}^{\text{rope weight}} + \overbrace{5 \text{ lb}}^{\text{bucket}} + \overbrace{8 \text{ lb} - \frac{1}{12} \frac{\text{lb}}{\text{ft}} \cdot y}^{\text{water weight}}$$

$$= 100 - 2y + 5 - \frac{1}{12}y = 105 - \frac{25}{12}y$$

$$\Rightarrow W = \int_0^{50} \left(105 - \frac{25}{12}y \right) dy$$

$$= \left[105y - \frac{25}{12} \cdot \frac{y^2}{2} \right]_0^{50} = 105 \cdot 50 - \frac{25}{24} \cdot 50^2 = \boxed{\frac{18,275}{6} \text{ lb} \cdot \text{ft}}$$

Extra scratch paper.

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