

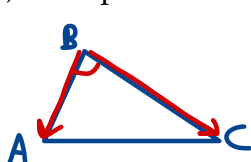
1. [4 points per part] For this problem, consider the points

$$A = (1, 4, -1)$$

$$B = (-1, 3, 1)$$

$$C = (5, 1, 4).$$

(a) Compute the angle $\angle ABC$.



$$\vec{BA} = \langle 2, 1, -2 \rangle \quad |\vec{BA}| = \sqrt{4+1+4} = 3$$

$$\vec{BC} = \langle 6, -2, 3 \rangle \quad |\vec{BC}| = \sqrt{36+4+9} = 7$$

$$\vec{BA} \cdot \vec{BC} = 12 - 2 - 6 = 4$$

$$\vec{BA} \cdot \vec{BC} = |\vec{BA}| |\vec{BC}| \cos \theta$$

$$4 = 3 \cdot 7 \cos \theta$$

$$\theta = \cos^{-1}\left(\frac{4}{21}\right) \approx 1.379 \text{ radians} \\ \approx 79.02^\circ$$

(b) Write parametric equations for the line through A which is parallel to the line $\overline{BC}$.

$$\text{Point: } A = (1, 4, -1)$$

$$\text{Direction: } \vec{BC} = \langle 6, -2, 3 \rangle$$

$$\begin{aligned} x &= 1 + 6t \\ y &= 4 - 2t \\ z &= -1 + 3t \end{aligned}$$

(c) Find the equation of the plane through A , B , and C .

$$\vec{BA} \times \vec{BC} = \langle 2, 1, -2 \rangle \times \langle 6, -2, 3 \rangle = \langle -1, -18, -10 \rangle$$

$$\text{Point: } (1, 4, -1) \quad -1(x-1) - 18(y-4) - 10(z+1) = 0$$

or

$$x + 18y + 10z = 63$$

2. [8 points] Find parametric equations for the line through $(0, 1, 2)$ which is perpendicular to the line $\mathbf{r}(t) = \langle 1 + 4t, 3t, 2 - t \rangle$ and parallel to the plane $x - 6y + 2z = 4$.

Orthogonal to $\langle 4, 3, -1 \rangle$

Also orthog. to $\langle 1, -6, 2 \rangle$

Direction vector: $\langle 4, 3, -1 \rangle \times \langle 1, -6, 2 \rangle = \langle 0, -9, -27 \rangle$ or, scaled down by $\frac{1}{9}$, $\langle 0, 1, 3 \rangle$

Through point $(0, 1, 2)$:

$$\begin{aligned} x &= 0 \\ y &= 1 + t \\ z &= 2 + 3t \end{aligned}$$

3. [3 points per point] For each of the following prompts, give an example of a vector function that meets the condition. (There are many possible answers! Just write one of them.)

You do not need to show any work.

- (a) A vector function whose space curve intersects the plane $z = 2$ exactly twice.

Lots of ways. One example:

$$\vec{r}(t) = \langle t, t, t^2 \rangle$$

↑ note: x- and y- coords at both intersections shouldn't be equal

- (b) A vector function whose normal component of acceleration is always 0.

Space curve should be a line, e.g. $\vec{r}(t) = \langle 0, t^3, 0 \rangle$

(or even just parametric eq's for a line)

- (c) A vector function whose space curve has a constant curvature of 7.

Easiest way: circle of radius $\frac{1}{7}$, like $\vec{r}(t) = \langle \frac{1}{7} \cos t, \frac{1}{7} \sin t, 0 \rangle$

4. [5 points per part] Let S be the surface $x^2 - 2x - y^2 - 4y + z^2 = -2$.

(a) Draw the trace of S in the plane $y = 1$. 

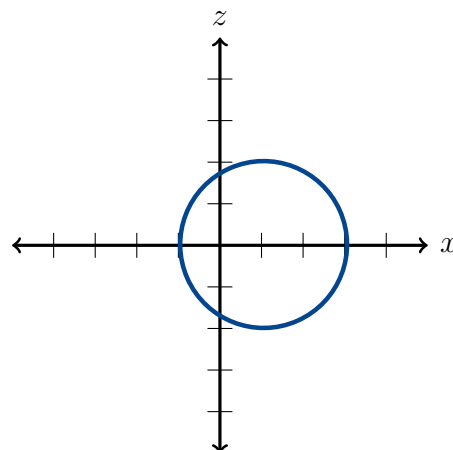
$$x^2 - 2x - 1 - 4 + z^2 = -2$$

$$x^2 - 2x + 1 + z^2 = 3 + 1$$

$$(x-1)^2 + z^2 = 4$$

Circle of radius 2

centered @ $x=1, z=0$



(b) Write the name of S . (Show your work!)

Note: this problem does not rely on part (a).

$$x^2 - 2x + 1 - y^2 - 4y - 4 + z^2 = -2 + 1 - 4$$

$$(x-1)^2 - (y+2)^2 + z^2 = -5$$

$$-\frac{(x-1)^2}{5} + \frac{(y+2)^2}{5} - \frac{z^2}{5} = 1$$

hyperboloid of 2 sheets

5. [6 points] Write a vector function whose space curve is the intersection of the surfaces

$$x^2 - y + 3z = 7$$

and

$$x^2 + z^2 = 4.$$

$$y = x^2 + 3z - 7$$

parametrize as a circle

$$x = 2\cos t$$

$$z = 2\sin t$$

$$y = 4\cos^2 t + 6\sin t - 7$$

$$\vec{r}(t) = \langle 2\cos t, 4\cos^2 t + 6\sin t - 7, 2\sin t \rangle$$

6. [5 points per part] Use the vector function $\mathbf{r}(t) = \langle t^2 - t, 2t^4, 1 - 4t \rangle$ to answer parts (a)–(c).

(a) Find parametric equations for the line tangent to the space curve of $\mathbf{r}(t)$ at $(6, 32, 9)$.

$$\vec{r}'(t) = \langle 2t - 1, 8t^3, -4 \rangle$$

$$\vec{r}'(-2) = \langle -5, -64, -4 \rangle$$

direction vector

$$\begin{aligned} 1 - 4t &= 9 \\ \downarrow \\ t &= -2 \end{aligned}$$

$$\begin{aligned} x &= 6 - 5t \\ y &= 32 - 64t \\ z &= 9 - 4t \end{aligned}$$

(b) Compute $\mathbf{T}(1)$, the unit tangent vector to the curve at $t = 1$.

$$\vec{r}'(1) = \langle 1, 8, -4 \rangle$$

$$|\vec{r}'(1)| = \sqrt{1 + 64 + 16} = 9$$

$$\vec{T}(1) = \frac{\vec{r}'(1)}{|\vec{r}'(1)|} = \left\langle \frac{1}{9}, \frac{8}{9}, \frac{-4}{9} \right\rangle$$

(c) A 3-kilogram object's position vector after t seconds is $\mathbf{r}(t)$ meters. Find the force applied to it at time $t = 1$.

$$\vec{a}(t) = \vec{r}''(t) = \langle 2, 24t^2, 0 \rangle$$

$$\vec{a}(1) = \vec{r}''(1) = \langle 2, 24, 0 \rangle$$

$$\vec{F}(1) = m\vec{a}(1) = 3\langle 2, 24, 0 \rangle = \langle 6, 72, 0 \rangle \text{ Newtons}$$