

Math 126 Midterm 2, Spring 2006 (Conroy) — Problem 1

The circle $x^2 + y^2 = \frac{1}{9}$ is $r = \frac{1}{3}$, so $\theta = 3$. With $r' = -\frac{1}{\theta^2} = -\frac{1}{9}$:

$$\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta} = \frac{-\frac{1}{9} \sin 3 + \frac{1}{3} \cos 3}{-\frac{1}{9} \cos 3 - \frac{1}{3} \sin 3} = \boxed{\frac{\sin 3 - 3 \cos 3}{\cos 3 + 3 \sin 3}} \approx -5.49.$$

Math 126 Midterm 2, Spring 2006 (Conroy) — Problem 2

The line has slope $\frac{1}{3}$. On the curve,

$$\frac{dy}{dx} = \frac{2t+2}{3t^2-3} = \frac{2}{3(t-1)} \quad (t \neq \pm 1).$$

Setting this to $\frac{1}{3}$ gives $t = 3$. ($t = -1$ makes both derivatives zero, so there is no tangent direction there.) The point is $\boxed{(18, 15)}$.

Math 126 Midterm 2, Spring 2006 (Conroy) — Problem 3

$\vec{r}' = \langle -\sin t, \cos t, m \rangle$, $\vec{r}'' = \langle -\cos t, -\sin t, 0 \rangle$, $|\vec{r}' \times \vec{r}''| = \sqrt{1 + m^2}$, $|\vec{r}'| = \sqrt{1 + m^2}$:

$$\kappa = \frac{1}{1 + m^2}, \quad \text{radius} = 1 + m^2 = 3 \Rightarrow \boxed{m = \sqrt{2}}.$$

Math 126 Midterm 2, Spring 2006 (Conroy) — Problem 4

$\vec{v} = \langle 2t, 1, 5 \rangle$, $\vec{a} = \langle 2, 0, 0 \rangle$, $|\vec{v}| = \sqrt{4t^2 + 26}$, $\vec{v} \times \vec{a} = \langle 0, 10, -2 \rangle$:

$$a_T = \frac{\vec{v} \cdot \vec{a}}{|\vec{v}|} = \frac{4t}{\sqrt{4t^2 + 26}},$$

$$a_N = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|} = \frac{2\sqrt{26}}{\sqrt{4t^2 + 26}}.$$

Math 126 Midterm 2, Spring 2006 (Conroy) — Problem 5

$|\vec{r}'| = |\langle 5, 2, 3 \rangle| = \sqrt{38}$, so $s = \sqrt{38}t$:

$$\vec{r}(s) = \left\langle \frac{5s}{\sqrt{38}} - 1, \frac{2s}{\sqrt{38}}, \frac{3s}{\sqrt{38}} + 2 \right\rangle.$$

Math 126 Midterm 2, Spring 2006 (Conroy) — Problem 6

(a)

$$f_y = x^2 + x \cos y - \frac{-2y}{x - y^2} = \boxed{x^2 + x \cos y + \frac{2y}{x - y^2}}.$$

(b)

$$f_{xy} = \frac{\partial}{\partial x} f_y = \boxed{2x + \cos y - \frac{2y}{(x - y^2)^2}}.$$