

Math 126 Midterm 2, Spring 2007 (Conroy) — Problem 1

$\vec{r}'(t) = \langle 4, 15t^2, 4t \rangle$, so $\vec{r}'(1) = \langle 4, 15, 4 \rangle$ with length $\sqrt{257}$.

(a) $\vec{T}(1) = \frac{1}{\sqrt{257}} \langle 4, 15, 4 \rangle$.

(b) Through $\vec{r}(1) = (4, 5, 2)$: $x = 4 + 4s, \quad y = 5 + 15s, \quad z = 2 + 4s$.

Math 126 Midterm 2, Spring 2007 (Conroy) — Problem 2

$x = r \cos \theta = (\sec \theta + \tan \theta) \cos \theta = 1 + \sin \theta$. So $x = 2$ would need $\sin \theta = 1$, i.e. $\theta = \frac{\pi}{2} + 2\pi k$, where $\sec \theta$ and $\tan \theta$ are undefined. Everywhere the curve is defined, $x < 2$ (it only approaches $x = 2$). **No**, it never meets the line $x = 2$.

Math 126 Midterm 2, Spring 2007 (Conroy) — Problem 3

$$\vec{v} = \langle 1, 2t, -t^{-2} \rangle, \vec{a} = \langle 0, 2, 2t^{-3} \rangle.$$

(a) At $t = 1$: $\vec{v} = \langle 1, 2, -1 \rangle$, $\vec{a} = \langle 0, 2, 2 \rangle$, so $a_T = \frac{\vec{v} \cdot \vec{a}}{|\vec{v}|} = \frac{2}{\sqrt{6}} = \boxed{\frac{\sqrt{6}}{3}}$.

(b) $\vec{v} \cdot \vec{a} = 4t - 2t^{-5} = 0 \Rightarrow t^6 = \frac{1}{2} \Rightarrow \boxed{t = \pm 2^{-1/6}}$.

Math 126 Midterm 2, Spring 2007 (Conroy) — Problem 4

$x' = 2t - 3$, $y' = 2t + 2$, $x'' = y'' = 2$:

$$\kappa = \frac{|x'y'' - y'x''|}{(x'^2 + y'^2)^{3/2}} = \frac{|2(2t - 3) - 2(2t + 2)|}{(8t^2 - 4t + 13)^{3/2}} = \frac{10}{(8t^2 - 4t + 13)^{3/2}}.$$

The curvature is largest when $8t^2 - 4t + 13$ is smallest, at $t = \frac{1}{4}$.

Math 126 Midterm 2, Spring 2007 (Conroy) — Problem 5

(a) Need $x + y > 0$: the open half-plane above the line $y = -x$ (the line itself excluded).

(b) $f_x = e^y - \frac{1}{x+y}$, so $f_{xy} = e^y + \frac{1}{(x+y)^2}$.