

Math 126 Midterm 2, Spring 2008 (Conroy) — Problem 1

$\vec{AB} = \langle 1, 2, 2 \rangle$, $\vec{AC} = \langle 5, 4, 3 \rangle$ from $A = (0, 1, 2)$; normal $\vec{AB} \times \vec{AC} = \langle -2, 7, -6 \rangle$:

$$-2x + 7(y - 1) - 6(z - 2) = 0 \Rightarrow \boxed{2x - 7y + 6z = 5}.$$

Math 126 Midterm 2, Spring 2008 (Conroy) — Problem 2

(a) Write the lines with parameters t and s : $1 + 2t = 19 + s$, $2 + t = 13 - \frac{s}{2}$. Then $s = 2t - 18$ and $2 + t = 22 - t$, so $t = 10$, $s = 2$; the z -check gives $33 = 43 - 10$ ✓. Intersection: $\boxed{(21, 12, 33)}$.

(b) The plane misses L when L is parallel to it (and not in it): $\langle a, 6, -7 \rangle \cdot \langle 2, 1, 3 \rangle = 2a - 15 = 0 \Rightarrow \boxed{a = \frac{15}{2}}$.
(The point $(1, 2, 3)$ gives $7.5 + 12 - 21 = -1.5 \neq 2$, so L is not in the plane.)

Math 126 Midterm 2, Spring 2008 (Conroy) — Problem 3

(a) $\vec{r}' = \langle 1, -2t, \frac{1}{t} \rangle \parallel \langle 2, -1, 8 \rangle$ requires $-2t = -\frac{1}{2}$ and $\frac{1}{t} = 4$: $t = \frac{1}{4}$. Point: $\left(\frac{1}{4}, \frac{63}{16}, -\ln 4\right)$.

(b) $x = \frac{1}{4} + 2s, \quad y = \frac{63}{16} - s, \quad z = -\ln 4 + 8s$.

Math 126 Midterm 2, Spring 2008 (Conroy) — Problem 4

$\vec{v} = \langle 1, 2t, -3t^{-4} \rangle, \vec{a} = \langle 0, 2, 12t^{-5} \rangle. \ a_T = 0 \iff \vec{v} \cdot \vec{a} = 4t - 36t^{-9} = 0 \iff t^{10} = 9:$

$t = \pm 3^{1/5} \approx \pm 1.246.$

Math 126 Midterm 2, Spring 2008 (Conroy) — Problem 5

$f_x = y^2 - 1$, $f_y = 2xy + 1$. $y = 1 \Rightarrow x = -\frac{1}{2}$; $y = -1 \Rightarrow x = \frac{1}{2}$. $f_{xx} = 0$, $f_{yy} = 2x$, $f_{xy} = 2y$, so $D = -4y^2 = -4 < 0$ at both:

$(-\frac{1}{2}, 1)$ and $(\frac{1}{2}, -1)$ are saddle points.

Math 126 Midterm 2, Spring 2008 (Conroy) — Problem 6

$\kappa = \frac{6|x|}{(1+9x^4)^{3/2}}$. For $x > 0$, the numerator of κ' is proportional to $6(1+9x^4) - 6x \cdot \frac{3}{2} \cdot 36x^3 = 6 - 270x^4$, zero at $x^4 = \frac{1}{45}$ (by symmetry also $x < 0$):

$$\boxed{(\pm 45^{-1/4}, \pm 45^{-3/4})} \approx (\pm 0.386, \pm 0.058).$$