

Math 126 Midterm 2, Winter 2006 (Conroy) — Problem 1

$\cos t = x$ and $\sin t = y + x$. Since $\cos^2 t + \sin^2 t = 1$:

$x^2 + (x + y)^2 = 1$

(i.e. $2x^2 + 2xy + y^2 = 1$).

Math 126 Midterm 2, Winter 2006 (Conroy) — Problem 2

At $t = e$: $x = \ln 1 = 0$, $y = 1 - 1 = 0$.

$$\frac{dx}{dt} = \frac{1}{t \ln t} = \frac{1}{e}, \quad \frac{dy}{dt} = \frac{1}{t} - \frac{2 \ln t}{t} = -\frac{1}{e}, \quad \frac{dy}{dx} = -1.$$

Tangent line: $y = -x$.

Math 126 Midterm 2, Winter 2006 (Conroy) — Problem 3

$z = 0$ needs $t = 0$ or $t = -10$; only $t = 0$ gives $x = 0, y = 1$. $\vec{r}'(t) = \langle 3t^2-1, 6t^5+2t, t+5 \rangle$, so $\vec{r}'(0) = \langle -1, 0, 5 \rangle$:

$$x = -s, \quad y = 1, \quad z = 5s.$$

Math 126 Midterm 2, Winter 2006 (Conroy) — Problem 4

$\kappa(x) = \frac{e^x}{(1 + e^{2x})^{3/2}}$. The numerator of κ' is $e^x(1 + e^{2x}) - 3e^{3x} = e^x(1 - 2e^{2x})$, which is zero when $e^{2x} = \frac{1}{2}$, i.e. $x = -\frac{\ln 2}{2}$ (κ' changes from + to - there):

$\left(-\frac{\ln 2}{2}, \frac{1}{\sqrt{2}}\right)$

Math 126 Midterm 2, Winter 2006 (Conroy) — Problem 5

$\vec{r}'(t) = \langle \sqrt{2}t^{1/2}, 1, t \rangle$, so $|\vec{r}'| = \sqrt{2t + 1 + t^2} = t + 1$:

$$L = \int_0^4 (t + 1) \, dt = 8 + 4 = \boxed{12}.$$

Math 126 Midterm 2, Winter 2006 (Conroy) — Problem 6

$\vec{r}'(0) = \langle -2, -1, 1 \rangle$ and $\vec{r}'' = \langle 1, 2, 2 \rangle$. Then $\vec{r}' \times \vec{r}'' = \langle -4, 5, -3 \rangle$, with length $\sqrt{50}$:

$$\kappa = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3} = \frac{5\sqrt{2}}{6\sqrt{6}} = \boxed{\frac{5\sqrt{3}}{18}} \approx 0.481.$$